

Inverse Scattering and Density Perturbations from Inflation

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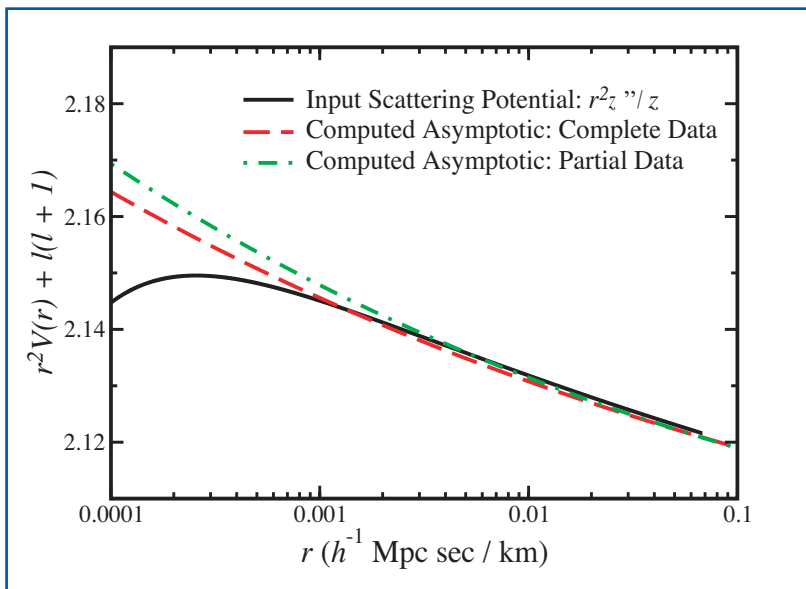
A wealth of data drives much of the current activity in cosmology. Amongst this data set, perhaps the cleanest signal from the primordial universe is the power spectrum of density fluctuations, which is obtained from observations of fluctuations in the microwave background sky and from observations of the large-scale clustering of matter. Especially in the case of microwave background observations, the observed fluctuations give direct information on the “initial conditions” for the density perturbations that manifest in the gravitational clustering of matter in the universe. In an inflationary universe, these initial density perturbations arise as a relic of quantum fluctuations from the very earliest times. The inflationary evolution causes these modes to grow in amplitude, with a time-evolution that is determined completely by the evolution of the space-time geometry, encoded in the scale-factor. This evolution imprints a signature of the inflationary evolution on the spectrum of modes.

Therefore, structure in the mode spectrum can be directly related to the evolution of the scale factor and therefore to the nature of the stress-energy which drives inflation. Because the ultimate nature of inflation is quite mysterious, information of this sort is very important for cosmology.

In the standard picture for quantum fluctuations and the generation of structure during inflation, the evolution of the gauge-invariant metric fluctuations is described by a second-order differential equation containing an effective potential that is determined by the evolving background. The form of this effective potential is different for scalar and tensor perturbations. For a specific inflationary model the effective potential and the evolution of the fluctuations can be calculated numerically. Then, in a second step, the related power spectra and the associated spectral indices can be obtained either numerically exactly or through approximation schemes [1]. The question arises if it is possible to find a solution to the inverse problem: given a power spectrum measured from the cosmic microwave background (CMBR) anisotropies and other observations, can we directly constrain the underlying inflationary equation of state? This question has led to the so-called inflationary reconstruction program. The goal of reconstruction is to determine as much information as possible about the stress-energy that drove the inflationary expansion. The reconstruction program splits naturally into two clearly defined tasks. The first task is to solve the linear problem, inverting from the observed power spectrum to the effective potential. The second task is to use the obtained effective potential to constrain the geometry and therefore to constrain the stress-energy tensor which drives inflation.

In recent work [2] we have shown how to use inverse scattering theory to complete the first task; we then showed how this information can be used in the second task, to constrain the physics of the inflationary epoch. The basic problem of calculating density perturbations is most naturally posed as a problem in scattering theory for the wave equation describing the evolution of gauge-invariant metric fluctuations. The wave solution is purely an incoming wave

Figure 1—
 Comparison of the reconstructed scattering potential to the input. The input was tabulated during computation of the power spectrum.



in the deep past, which follows from the positive-energy vacuum condition. Solutions that obey this condition are important in scattering theory and are called Jost solutions. The behavior of the Jost solution is described by the Jost-function, and it encodes essentially all the information about the scattering theory. It is possible to directly connect the Jost-function and the basic function of interest, the power spectrum itself. At this point the full machinery of inverse scattering theory can be invoked, which allows us to reconstruct the effective potential from the Jost function, by solving a linear integral equation. Rather than solving this numerical problem, we have shown instead [2] how the Jost function encodes information about the asymptotic behavior of the effective potential, in a manner which is directly applicable to the cosmological inversion problem.

Instead of explaining technical details we will demonstrate our idea with a concrete example. Consider the power spectrum in Fig. 1. This power spectrum was computed numerically from an inflation model with a ϕ^2 potential. Computations of power spectra for other single-field models show that the properties of this power-spectrum are generic for a large class of such models and it serves as a good illustration. The first step is to extract the leading power-law behavior in this power spectrum, shown in Fig. 1 in red. Having obtained the remaining function it is now possible to reconstruct the asymptotic behavior for the effective potential. In Fig. 2 we compare the reconstructed function to the input which gave the power spectrum of Fig. 1. Note that both reconstructions agree well with the input and with each other for large r (the time variable r is zero at the end of inflation and very large in the far past), as they should. A small additive error results from lack of precision in the determination of the power-law function we fit, which determines the pedestal for each of the functions. The input function rolls over for small r , which indicates the end of inflation; of course, this rollover is not captured by the asymptotic reconstruction. The procedure we

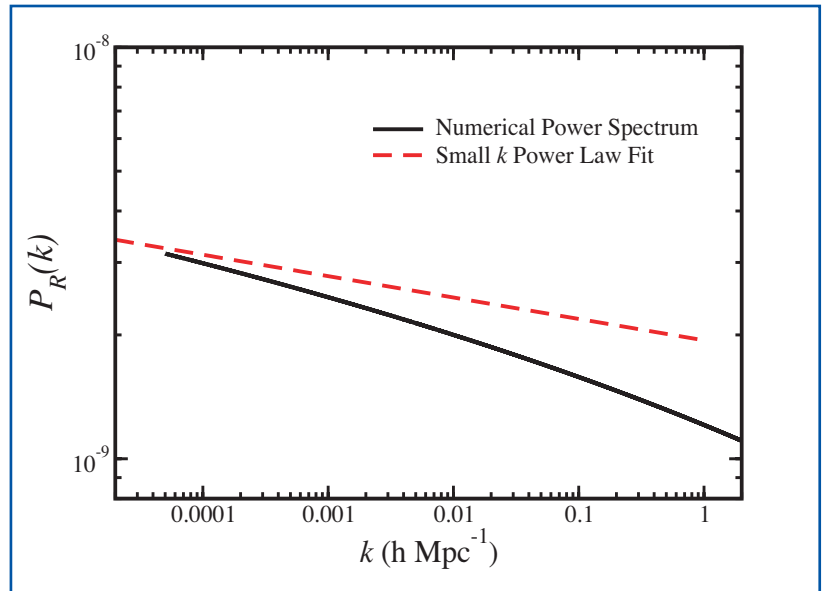


Figure 2—
Power spectrum
computed for a ϕ^2
inflation model. The
dashed line indicates
the small k behavior,
a pure power law.

have described is essentially nonparametric. No particular model for the stress-energy need be assumed; geometric information is extracted directly from the power spectrum. The relations of scattering theory provide a very convenient general framework for posing the density-perturbation problem in inflationary cosmology. We have seen that inflationary reconstruction is essentially a problem in inverse scattering theory. Direct relations were used to invert from the power spectrum to asymptotic quantities of interest, which describe the background evolution through most of the inflationary epoch.

[1] S. Habib, A. Heinen, K. Heitmann, and G. Jungman, “Precision Cosmology: Scalar and Tensor Perturbations from Inflation,” contribution on p. 94 in this volume and references therein.

[2] S. Habib, K. Heitmann, and G. Jungman, LA-UR-04-6667 (2004), *Phys. Rev. Lett.* (in press).

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